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# A Tracking Accuracy Guarantee System on Augmented Reality 

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#### Abstract

Improvement of tracking accuracy is an important issue when applying augmented reality to nuclear power plant fieldwork. Tracking accuracy depends highly on the marker arrangement when employing a tracking method using a camera and markers. For those reasons, this study develops a wheel tracking error computation method to compute the tracking error from the marker arrangement and errors in the screen coordinate. An evaluation experiment was conducted using a kind of linecode markers developed before. Experimental results show that the tracking error computation is reliable and the speed of the tracking error computation is affordable to be applied in real time error estimation in NPP field work support.


Keywords: Augmented reality, tracking accuracy, tracking error estimation method, Marker Arrangement, design system, evaluation method, industrial plant, maintenance support, linecode marker

## 1. INTRODUCTION

In Japan, nuclear power plants (NPPs) must undergo maintenance every 13 months. The NPPs' operation must be halted for about one month to disassemble and check equipment during the maintenance period. The NPP components are, however, much more numerous than those of a thermal power plant of similar scale. Moreover, NPP maintenance work is much harder because the NPP structure is typically more complicated. In addition, lack of expert field workers has become a widespread problem. Simultaneous improvement of maintenance work efficiency and reduction of human error are desired to improve NPPs' competitive power after the impending liberalization of the electric market.

Augmented reality (AR) technology is broadly used for entertainment, education, surgery simulations, and other applications [1]. AR offers great possibilities to support NPP fieldwork. Using AR technique, we can identify 2D and 3D positions more intuitively than when using a legacy interface such as paper instruction documents. For example, annotations, informative figures, 3D maps, and videos can be superimposed in the field worker's view. It is expected that the workers can work more accurately and rapidly using AR support.

An accurate and reliable tracking method is indispensable to apply AR technology to NPP fieldwork. To date, many tracking methods have been developed. Tracking technologies, such as global positioning systems, infrared rays, stereo view cameras, and ultrasonic and magnetic fields are popular now[1]. Although these tracking methods can realize accurate tracking at a certain level, they are difficult to apply to NPP fieldwork because these methods do not meet all requirements from the viewpoints of accuracy,
stability, economics, and availability for the NPP field work[2][3]. Consequently, we sought to apply tracking methods that use cameras and markers (marker-based tracking) to NPP fieldwork. To achieve high reliability on tracking, we will develop a high reliability tracking method in this study with assurance with tracking error estimation (TEE). When tracking error is larger than a certain threshold, user will be warned because the tracking accuracy is not reliable at this time.

According to the high reliability requirement of NPP field work support, the TEE algorithm should satisfy the following requirements.
(1) Find the largest possible tracking error rather than probabilistic average error.
(2) It must be sufficiently fast for practical use.

Traditional TEE algorithms, such as [4], can not meet these requirements. Therefore, a wheel TEE algorithm is developed to meet these requirements.

## 2. THE WHEEL TRACKING ERROR COMPUTATION ALGORITHM

### 2.1 Definitions and Settings

Figure 1 shows the coordinate definitions used for this study. Screen coordinates are defined as the origin at the center of the screen $D$; the x axis extends left to right, and the $y$ axis extends upward on the screen. The camera coordinate is defined as the origin at the camera's focus point C, the z direction to the line of sight of the camera. The $x$ and $y$ directions of the camera coordinate are the same as the screen coordinate.


Fig. 1 Coordinate system.
The camera pose is a six-dimensional vector including the camera's three-dimensional (3D) position and 3D rotation.

Pose C' satisfies that the camera at C' can capture the same markers as the camera at $C$. Here the following variables are used: $Q_{i}, i=1,2 \ldots n$ are the 3D position of visible markers in the camera coordinate; $n$ is the number of visible markers; $m_{i}$, a 2D vector, is the image point of $Q_{i}$ in pose $C$ screen coordinate; $m_{i}^{\prime}$ is the image point of $Q_{i}$ in pose C' screen coordinate. The image difference of $Q_{i}$ between the camera at $C$ and that at $C^{\prime}$ is defined as $\left\|\mathrm{m}_{\mathrm{i}}-\mathrm{m}_{\mathrm{i}}^{\prime}\right\|$. The largest image difference (LID) between the camera at C and that at $\mathrm{C}^{\prime}$ is defined as the largest image difference among any two correspondent marker pairs, i.e. $m_{i}$ and $m^{\prime}$, in both images captured by camera at C and at $\mathrm{C}^{\prime}$. Equation (1) shows the definition of LID.

$$
\begin{equation*}
\operatorname{LID}\left(\mathrm{C}^{\prime}, \mathrm{C}, \mathrm{M}\right)=\max _{\mathrm{i}=1,2, \ldots, \mathrm{n}}\left\|\mathrm{~m}_{\mathrm{i}}-\mathrm{m}_{\mathrm{i}}^{\prime}\right\| \tag{1}
\end{equation*}
$$

In that equation, $M$ is an $M A$. The marker's position accuracy on the screen is always limited because the camera resolution is limited when the marker is recognized using image processing. The position accuracy is assumed to be $\Delta$. When $\operatorname{LID}\left(\mathrm{C}^{\prime}, \mathrm{C}, \mathrm{M}\right) \leq \Delta$, the pose of $\mathrm{C}^{\prime}$ and C can not be distinguished and we say that C and $\mathrm{C}^{\prime}$ can capture the same image. The tracking error region (TER) is a set of points at which a camera can capture the same image as the real pose C , where $\mathrm{C}^{\prime}$ is one pose in the TER.

### 2.2 Computation of Limitation Circles

Next, the algorithm for computing TER will be described. Presuming that n markers are captured by the camera ( n should be larger than 3 for tracking $¥$ cite\{p3p\}), compute TER as the intersection region of following areas in the $\mathrm{x}-\mathrm{z}$ plane: (1) for each 2 markers among the $n$ markers, the area in which the camera can capture the same x-coordinates of the two markers; (2) for each single marker, the area in which the camera can capture the same $y$-coordinates of the marker. These areas are enclosed by circles. These circles are called limitation circles.

### 2.2.1 Limitation Circles for the x Coordinate

In Fig. 2, $P_{A}$ and $P_{B}$ are projection points of $Q_{A}$ $\left(x_{a}, y_{a}, z_{a}\right)$ and $Q_{B}\left(x_{b}, y_{b}, z_{b}\right)$ on the $x-z$ plane in camera coordinate. $A$ is the intersection point
between $\mathrm{CP}_{\mathrm{A}}$ and the screen x axis. B is the intersection point between CP_B and the screen x axis. $A, A_{1}, A_{2}, B, B_{1}, B_{2}$ are on the screen $x$ axis and set $\left\|\mathrm{A}_{1} \mathrm{~A}\right\|=\left\|\mathrm{AA}_{2}\right\|=\left\|\mathrm{B}_{2} \mathrm{~B}\right\|=\left\|\mathrm{BB}_{1}\right\|=\Delta$. We cannot distinguish $A_{1}, A_{2}$ from $A$ and $B_{1}, B_{2}$ from $B$ because their differences in screen coordinate are smaller than $\Delta$. Consequently, pose $\mathrm{C}^{\prime}$, which can capture same $x$ dimension images of $P_{A}$ and $P_{B}$ as the real camera pose, satisfies

$$
\begin{equation*}
\alpha_{2} \leq \angle P_{A} C^{\prime} P_{B} \leq \alpha_{1} \tag{2}
\end{equation*}
$$



Fig. 2 Computation method for the track of $\mathrm{C}^{\prime}$ : the camera at $\mathrm{C}^{\prime}$ can capture the same images of $Q_{A}$ and $Q_{B}$ in the $x$ coordinate.

Here $\alpha_{2}=\angle A_{2} C B_{2}$ and $\alpha_{1}=\angle A_{1} C B_{1}$. They can be computed from

$$
\begin{align*}
& \tan \angle A_{1} C D=\tan \angle A C D+\frac{\left\|A_{1} A\right\|}{f}=\frac{x_{a}}{z_{a}}+\frac{\Delta}{f}  \tag{3}\\
& \tan \angle A_{2} C D=\tan \angle A C D-\frac{\left\|A_{2} A\right\|}{f}=\frac{x_{a}}{z_{a}}-\frac{\Delta}{f}  \tag{4}\\
& \tan \angle B_{1} C D=\tan \angle B C D+\frac{\left\|B_{1} B\right\|}{f}=\frac{x_{b}}{z_{b}}+\frac{\Delta}{f}  \tag{5}\\
& \tan \angle B_{2} C D=\tan \angle B C D-\frac{\left\|B_{2} B\right\|}{f}=\frac{x_{b}}{z_{b}}-\frac{\Delta}{f} \tag{6}
\end{align*}
$$

where D is the center of the screen and $f$ is the focal length of the camera. The track of $\mathrm{C}^{\prime}$ on $\angle P_{A} C^{\prime} P_{B}=\alpha_{1}$ and $\angle P_{A} C^{\prime} P_{B}=\alpha_{2}$ are circles 1 and 2, as shown in Fig. 3. The $\mathrm{C}^{\prime}$ that satisfies (2) should be in the gray area between the two circles.


Fig. 3 Computation method of a limitation circle
The computation methods for circles 1 and 2
are the same. Here we take circle 1 for description. First assume that F is on circle 1 and satisfies $\left\|\mathrm{P}_{\mathrm{A}} \mathrm{F}\right\|=\left\|\mathrm{P}_{\mathrm{B}} \mathrm{F}\right\|$ to compute the center of circle 1, and that $E$ is on $P_{A} P_{B}$ and $\left\|P_{A} E\right\|=\left\|P_{B} E\right\|$. Because $\left\|\mathrm{P}_{\mathrm{A}} \mathrm{F}\right\|=\left\|\mathrm{P}_{\mathrm{B}} \mathrm{F}\right\|$, EF is midnormal of $\mathrm{P}_{\mathrm{A}} \mathrm{P}_{\mathrm{B}}$. Therefore, the center of circle 1 , point $0\left(\mathrm{x}_{0}, 0, \mathrm{z}_{0}\right)$, is on EF . Because $\|O \mathrm{~F}\|=\left\|\mathrm{OP}_{\mathrm{B}}\right\|=\mathrm{R}$ and $\angle \mathrm{P}_{\mathrm{A}} \mathrm{FP}_{\mathrm{B}}=$ $\alpha_{1}$ ( R is the radius of circle 1 ), $\angle \mathrm{OFP}_{\mathrm{B}}=\angle \mathrm{OP}_{\mathrm{B}} \mathrm{F}$ $=\alpha_{1} / 2$. Therefore,

$$
\begin{equation*}
R=\frac{\left\|P_{A} E\right\|}{\sin \angle E O P_{B}}=\frac{\left\|P_{A} P_{B}\right\|}{2 \sin \alpha_{1}} \tag{7}
\end{equation*}
$$

The slope angle of $\mathrm{P}_{\mathrm{A}} \mathrm{P}_{\mathrm{B}}$ is $\gamma=\arctan \frac{z_{b}-z_{a}}{x_{b}-x_{a}}$; the slope angle of $\mathrm{P}_{\mathrm{A}} \mathrm{O}$ is $\theta=\alpha_{1}+\gamma-\pi / 2$. Therefore,

$$
\begin{align*}
& x_{O}=x_{a}+R \cos \theta  \tag{8}\\
& y_{O}=y_{a}+R \sin \theta \tag{9}
\end{align*}
$$

The gray area in Fig. 3 is called the x limitation area. We can compute the $x$ limitation area for any two markers in the n markers as for $\mathrm{Q}_{\mathrm{A}}$ and $\mathrm{Q}_{\mathrm{B}}$.
2.2.2 Limitation Circles for the $y$ Coordinate

Consider the $y$ direction in the screen coordinate, as shown in Fig. 4. Hereby, set $\left\|\mathrm{A}_{1} \mathrm{~A}\right\|=\left\|\mathrm{AA}_{2}\right\|=\Delta, \mathrm{CP}_{\mathrm{A}}$ as the projection line of $C Q_{A}$ on the $\mathrm{x}-\mathrm{z}$ plane and $\mathrm{A}_{2} \mathrm{~A}_{1}$ is parallel to y axis of the screen coordinate. $D$ is the center of the screen and $\|C D\|=f$. $E$ is a projection point of $A$ on the $x$ axis of the screen coordinate.


Fig. 4 Computation Method for Track of $C^{\prime}$ : the camera at C' can capture the same images of Q_A in the y coordinate.

Consequently, C', which can capture the same image of Q_A in y direction as C , should satisfy

$$
\begin{equation*}
\beta_{2} \leq \angle Q_{A} C^{\prime} P_{A} \leq \beta_{1} \tag{10}
\end{equation*}
$$

Hereby $\quad \beta_{2}=\angle A_{2} C P_{A} \quad$ and $\beta_{1}=\angle A_{1} C P_{A}$. Centers of both circles are $P_{A}$. Radii of the circles can be computed as $\left\|Q_{A} P_{A}\right\| \operatorname{ctan} \beta_{2}$ and $\left|\left|Q_{A} P_{A}\right|\right| \operatorname{ctan} \angle \beta_{1}$.

$$
\begin{align*}
& \text { Here, } \\
& \beta_{2}=\operatorname{atan}\left(\tan \angle Q_{A} C P_{A}-\Delta \cos \angle D C E\right) \\
& \beta_{1}=\operatorname{atan}\left(\tan \angle Q_{A} C P_{A}+\Delta \cos \angle D C E\right)  \tag{12}\\
& \angle D C E=\operatorname{atan}\left(x_{a} / z_{a}\right) \tag{13}
\end{align*}
$$

The area (a ring) between the two circles is called the $y$ limitation area. Compute y limitation areas for any marker in the n markers as for $Q_{A}$.

### 2.3 Computation of Tracking Error Region and Tracking Error

The intersection region of all the x and y limitation areas is TER because the camera can capture all the markers exactly as the real pose. The computation method for the TER is shown in Fig. 4. Here, to introduce the computation method easily, we use a three-marker MA. For a n-marker MA, the computation method is the same.


Fig. 4 Sample image of tracking error region.
The computation method is called the wheel TEE algorithm. In the following, we will explain the TER computation procedure. All of $Q_{A}, Q_{B}$, and $Q_{C}$ are respectively projected to $P_{A}, P_{B}$ and $P_{C}$ in the x-z plane of the camera coordinate. As shown in Fig. 4, the camera at any position of Area 1 (x limitation area of $Q_{A}$ and $Q_{B}$ ) can view the same x-dimension images of $Q_{A}$ and $Q_{B}$; the camera at any position of Area 2 (y limitation area of $Q_{A}$ ) can view the same y image of $Q_{A}$. Compute x limitation areas for $\left(Q_{B}, Q_{C}\right)$ and $\left(Q_{A}, Q_{C}\right)$ similarly as that for $\left(Q_{A}, Q_{B}\right)$. Compute $y$ limitation areas for $Q_{B}$ and $Q_{C}$ similarly as that for $Q_{A}$. The camera can view the same images of the three markers in the intersection region of all of the above regions: this intersection region is the TER.

We can compute the edges of the TER as a set of arcs. The most distant position in the TER from the real camera must be at one intersection point of the circles introduced above (proved in section 2.4). We must compute two circles to generate an x limitation area for each two markers and compute two circles to generate a y limitation area for each marker when the MA has $n$ markers. We must compute $n(n+1)$ circles and $n(n+1) / 2$ limitation areas for $n$ markers. Then compute the intersection points of these circles, find the farthest intersection point which can be in all limitation areas. The distance between the point and C is the tracking error.

The above method can compute tracking error on x-z plane as $e_{x z}$. By rotating the MA through z
axis, tracking error of $y$ direction can also be computed by above algorithm as $e_{y z}$. In the evaluation experiment, tracking error $e$ is $\max \left(e_{x z}, e_{y z}\right)$.

### 2.4 Why Computing only Intersection Points

We need to prove that the farthest point in the TER is one intersection point of all the limitation circles. Also, the most distant point in any intersection arc to $C$ is one of the ends of the arc.


Fig. 5 Proof for the largest point is at one end of arc.

As shown in Fig. 5, C is the origin of the camera coordinate. One limitation circle 0 is at $\left(x_{0}, 0, z_{0}\right)$ with radius of R. Also, $z_{0}>0$ because marker(s) that generated the circle 0 should be visible to the camera. D and E are two intersection points between circle $O$ and other circles. Now we assume that $\widehat{D E}$ is one edge of the TER, and that K is the most distant point from C on circle 0 and $\|C K\|=\|C O\|+R$. We must prove that the greatest distance from $C$ to any point on $\widehat{D E}$ is either $\|C D\|$ or $\|C E\|$. The problem can be described as

$$
\begin{equation*}
\max f(t)=x^{2}+z^{2} \tag{14}
\end{equation*}
$$

where

$$
\begin{align*}
& x=x_{0}+R \cos (t)  \tag{15}\\
& z=z_{0}+R \sin (t)  \tag{16}\\
& \text { strict to } \\
& t \in\left[t_{D}, t_{E}\right] \subset\left[\frac{\pi}{2}-\operatorname{atan} \frac{x_{0}}{z_{0}}, \frac{5 \pi}{2}-\operatorname{atan} \frac{x_{0}}{z_{0}}\right] . \tag{17}
\end{align*}
$$

Equation (17) includes the assumption that $\notin \widehat{D E}$. Because (1) there is at least one marker on circle 0 or at 0 and (2) $\widehat{D E}$ is one edge of the TER, if $\in \widehat{D E}$, then the tracking error is equal or greater than $\|C K\|$. This tracking error is larger than the distance between the marker and the camera. Therefore, this kind of MA is not feasible and must be modified. The case in which $K \in \widehat{D E}$ can be determined and eliminated by checking whether $t_{K} \in\left(t_{D}, t_{E}\right)$. Therefore, we need consider only the case for which $K \notin \widehat{D E}$.

Substitute (15) (16\}) into (14):
$\max f\left(\mathrm{t}^{\prime}\right)=\mathrm{K}_{1}+\mathrm{K}_{2} \sin \left(\mathrm{t}^{\prime}\right)$
where $t^{\prime}=t+\operatorname{atan}\left(x_{0} / z_{0}\right), \quad K_{1}=x_{0}^{2}+z_{0}^{2}+$ $R^{2}, \quad K_{2}=2 R \sqrt{x_{0}^{2}+z_{0}^{2}}, \quad t_{D}^{\prime}=t_{D}+\operatorname{atan}\left(x_{0} /\right.$ $\left.z_{0}\right), t_{E}^{\prime}=t_{E}+\operatorname{atan}\left(x_{0} / z_{0}\right)$.
We prove the problem by contradiction. Assume that $\exists t^{\prime} \in\left(t_{D}^{\prime}, t_{E}^{\prime}\right)$ satisfies $f\left(t^{\prime}\right)>f\left(t_{D}^{\prime}\right)$ and
$f\left(t^{\prime}\right)>f\left(t_{E}^{\prime}\right)$. From (18),
$f\left(t^{\prime}\right)-f\left(t_{D}^{\prime}\right)=2 K_{2} \sin \frac{t^{\prime}-t_{D}^{\prime}}{2} \cos \frac{t^{\prime}+t_{D}^{\prime}}{2,}>0$
$f\left(t^{\prime}\right)-f\left(t_{E}^{\prime}\right)=2 K_{2} \sin \frac{t-t_{E}}{2} \cos \frac{t^{\prime}+t_{E}}{2}>0$
Because $\sin \frac{t^{\prime}-t_{D}^{\prime}}{2}>0, \quad \sin \frac{t^{\prime}-t_{E}^{\prime}}{2}<0$ and $K_{2}>0$,

$$
\begin{equation*}
\cos \frac{t^{\prime}+t_{D}^{\prime}}{2}>0, \quad \cos \frac{t^{\prime}+t_{E}^{\prime}}{2}<0 . \tag{21}
\end{equation*}
$$

Furthermore, $\frac{t^{\prime}+t_{D}^{\prime}}{2,}, \frac{t^{\prime}+t_{E}}{2} \in\left[\frac{\pi}{2}, \frac{5 \pi}{2}\right]$. Therefore,
$\frac{t^{\prime}+t_{D}^{\prime}}{2}>\frac{3 \pi}{2}>\frac{t^{\prime}+t_{\mathrm{E}}^{\prime}}{2}$.
This contradicts $t_{E}^{\prime}>t_{D}^{\prime}$. Therefore, $f\left(t^{\prime}\right)<$ $f\left(t_{D}^{\prime}\right)$ or $f\left(t^{\prime}\right)<f\left(t_{E}^{\prime}\right)$, which means that $\|\mathrm{CD}\|$ or ||CE\| is the longest distance from C to any point on $\widehat{D E}$.

## 3. EVALUATION EXPERIMENT

### 3.1 Linecode Marker and its Tracking Method

In this section, we present the shape design of a linecode marker and the linecode parameters (marker size, distance between markers, etc.), along with a determination method for the tracking range.


Fig. 6 Conceptual image of linecode marker
Fig. 6 shows a conceptual illustration of the linecode marker. The linecode marker is a combination of black elements: each element corresponds to one bit. The square element signifies " 0 ". The double-sized rectangle element denotes "1".

In this study, bit number $\mathrm{n}=8$ and $\mathrm{s}=30 \mathrm{~mm}$.
The tracking method is a traditional P3P $¥$ cite $\{\mathrm{p} 3 \mathrm{p}\}$ method with least square optimization for other feature points except the used 3 feature points. Only the top elements and bottom elements of the linecode markers are used for tracking $¥$ cite[linecode].

## $3.2 \Delta$ Computation

$\Delta$ is computed from two types of screen errors, error in recognition $\Delta_{r}$ and error in tracking $\Delta_{t}$.

Here $\Delta_{r}$ is computed from the characteristics of the linecode marker. $\Delta_{t}$ can be obtained from the tracking method. $\Delta_{r}$ can be computed from perpendicular recognition error $\Delta_{p}$ and inline recognition error $\Delta_{i}$.
$\Delta_{p}$ is defined as the distance between the center of the selected element (the top element or the bottom element) and the least square line of all the elements of the linecode marker.

The distance between the top and bottom element can be computed as $\left(n \sum c_{i}+n-2-\right.$
$\left.\frac{c_{0}+c_{1}}{2}\right) s$, where $c_{i}$ the code of the ith element in the linecode marker.

$$
\begin{align*}
& \Delta_{i} \text { is computed as follows, } \\
& d_{i}=\left(i \sum_{j=0}^{i-1} c_{j}+i-2-\frac{c_{0}+c_{i-1}}{2}\right) s  \tag{23}\\
& \Delta_{i}=\max _{i}\left|\sqrt{\left(x_{0}-x_{i}\right)^{2}+\left(y_{0}-y_{i}\right)^{2}}-d_{i}\right| \tag{24}
\end{align*}
$$

Here, $\left(x_{i}, y_{i}\right)$ is the screen position and ( $x_{i}^{\prime}, y_{i}^{\prime}$ ) is the projected screen position of the ith selected feature points. k is two times of the number of the linecode markers.
$\Delta_{t}$ is defined as the difference between the feature points on the screen and the projected feature points computed from the tracking result pose[2].

$$
\begin{equation*}
\Delta_{t}=\max _{i} \sqrt{\left(\mathrm{x}_{\mathrm{i}}-\mathrm{x}_{\mathrm{i}}^{\prime}\right)^{2}+\left(\mathrm{y}_{\mathrm{i}}-\mathrm{y}_{\mathrm{i}}^{\prime}\right)^{2}} \tag{25}
\end{equation*}
$$

At last the $\Delta$ is decided to be
$\Delta=\Delta_{r}+\Delta_{t}$
where $\Delta_{r}=\sqrt{\Delta_{p}^{2}+\Delta_{i}^{2}}$ because these two error are perpendicular to each other.

### 3.3 Experimental Configuration

Figure 7 portrays the experiment configuration. The global coordinate is defined as the origin at 0 in Fig. 7, the x direction is set toward the reader, the $y$ direction is set to up, and the $z$ direction is set to the left. All markers are placed vertically. The placement error is within 1 cm . This configuration is adopted to simulate the complex and large-scale environment in the NPP field. The camera is placed before the markers and the camera is moved from 0 to 8 m in the x direction and -2 to -11 m in the z direction. The camera was rotated at every point with $0,20,40$ degrees (to the direction in which markers can be captured). The average illumination condition in the room is 1050 lux. No linecode marker-like articles exist in the experiment room.


Fig. 7 Experimental setup (unit: m).

### 3.4 Experimental Results on Accuracy

The real tracking error when the camera is placed at 0,20 , and 40 degrees are shown from left to right in Fig. 8. The estimated tracking error is shown in Fig. 9. The distance error is illustrated using the circle's diameter and grayscale. The circle's diameter is fixed and the error (unit: mm) is drawn on the circle if the error is greater than 40 cm .

The result shows that the wheel TEE algorithm can compute tracking error correctly. The computation time is in 1 ms and therefore it is possible to implement the wheel TEE algorithm into the tracking method in real-time.

## 4. CONCLUSION

A wheel TEE algorithm is developed and evaluated in long distance tracking. The result shows the wheel TEE algorithm is feasible for implementing a high reliability tracking method.

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Fig. 8 Real tracking error: from left to right, camera is set at $0^{\circ}, 20^{\circ}, 40^{\circ}$


Legend
Not Enough Markers
e $<50 \mathrm{~mm}$
$50 \mathrm{~mm}<\mathrm{e}<100 \mathrm{~mm}$
$100 \mathrm{~mm}<\mathrm{e}<150 \mathrm{~mm}$ $150 \mathrm{~mm}<e<200 \mathrm{~mm}$ $200 \mathrm{~mm}<e<250 \mathrm{~mm}$ $250 \mathrm{~mm}<e<300 \mathrm{~mm}$ $300 \mathrm{~mm}<\mathrm{e}<400 \mathrm{~mm}$ e $>400 \mathrm{~mm}$

Fig. 9 Estimated tracking error: from left to right, camera is set at $0^{\circ}, 20^{\circ}, 40^{\circ}$

